Gaussian Quadrature

$$\int_{-1}^{1} f(x)\,dx = \sum W_i\, f(x_i)$$

Two points	Three points
x_1 , x_2	x_1 , x_2 , x_3
w_1 , w_2	w_1 , w_2 , w_3
$w_1 = 1$ $\quad x_1 = \frac{-1}{\sqrt{3}}$	$w_1 = \frac{5}{9}$ $\quad x_1 = -\sqrt{\frac{3}{5}}$
$w_2 = 1$ $\quad x_2 = \frac{1}{\sqrt{3}}$	$w_2 = \frac{8}{9}$ $\quad x_2 = 0$
	$w_3 = \frac{5}{9}$ $\quad x_3 = \sqrt{\frac{3}{5}}$

Ex: Use three points to get

$$\int_{-1}^{1} \frac{2}{\sqrt{x+4}}\,dx$$

$$I = \frac{5}{9} f\left(-\sqrt{\frac{5}{3}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{5}{3}}\right) = 2.016$$

Important

$$\int_a^b f(x)\,dx \qquad if\ a \neq -1, b \neq 1$$

$$\int_a^b f(x)\,dx \rightarrow \int_{-1}^{1} f(z)\,dz$$

$$\frac{x-a}{b-a} = \frac{z+1}{2} \qquad dx = \frac{b-a}{2}\,dz$$

Ex: Use three points to get

$$\int_0^1 \ln(3 + sin(x))\ dx$$

$$x = \frac{z+1}{2} \qquad dx = \frac{1}{2}\,dz$$

$$\int_0^1 \ln(3 + sin(x))\ dx \rightarrow f(z) = \int_{-1}^{1} \frac{1}{2} ln(3 + sin\frac{z+1}{2})\ dz$$

$$I = \frac{5}{9} f\left(-\sqrt{\frac{5}{3}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{5}{3}}\right) = 1.2385$$